Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Partial Marks
1(a)	$3x^2 - 15x + 12$ [* 0] oe where * is any inequality sign or =	**B1**	
	Factorises or solves *their* 3-term quadratic	**M1**	**FT** *their* 3-term quadratic
	$x < 1$ or $x > 4$ mark final answer	**A1**	
1(b)(i)	$3(x-2)^2 + 4$	**3**	**B2** for $3(x-2)^2$ or **B1** for $(x-2)^2$ or $a = 3$, $b = -2$ and **B1** for $a(x+b)^2 + 4$ with numerical values of a and b or $c = 4$
1(b)(ii)	$y =$ *their* 4	**B1**	**STRICT FT** *their* 4 from part **(i)**

Question	Answer	Marks	Partial Marks
2(a)	$\dfrac{dy}{dx} = 64x - \dfrac{2x^{-3}}{8}$ oe, isw	B2	**B1** for $\dfrac{dy}{dx} = 64x + kx^{-3}$ or $\dfrac{dy}{dx} = kx - \dfrac{2x^{-3}}{8}$ where k is a non-zero constant or **SC1** for $\dfrac{dy}{dx} = 64x - \dfrac{2}{8}x^{-3} + c$
	their $\dfrac{dy}{dx} = 0$ and attempt to solve	M1	**FT** *their* derivative providing it has two terms and at least one term is a correct power of x
	(0.25, 4), (–0.25, 4) nfww, isw	A2	**A1** for either stationary point correct or for $x = \pm\ 0.25$ nfww or, if $\dfrac{dy}{dx} = 64x - 16x^{-3}$, then award **SC2** for $\left(\pm\dfrac{1}{\sqrt{2}}, \dfrac{65}{4}\right)$ oe or **SC1** for either of these stationary points or $x = \pm\dfrac{1}{\sqrt{2}}$ oe

Question	Answer	Marks	Partial Marks
2(b)	Correct second derivative: $\frac{d^2y}{dx^2} = 64 + \frac{3}{4}x^{-4}$ oe, isw	**M1**	**FT** *their* $\frac{dy}{dx} = mx + nx^{-3}$ where $m \neq 0$ and $n \neq 0$ seen in part (a)
	$64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(\frac{1}{4}\right)^{-4} > 0$ or when $x = 0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe **and** minimum [points] oe OR $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} = 256$ or $64 + \frac{3}{4}\left(-\frac{1}{4}\right)^{-4} > 0$ or when $x = -0.25$ $\frac{d^2y}{dx^2} = 256$ or $\frac{d^2y}{dx^2} > 0$ oe **and** minimum [points] oe OR $\frac{d^2y}{dx^2} = 64 + \frac{3}{4x^4}$ and this is positive for any value of x **and** minimum [points]	**A2**	**dep** on $x = \pm 0.25$ nfww in part **(a)** **A1 dep** on $x = 0.25$ or $x = -0.25$ nfww in part (a) for correctly showing or stating $\frac{d^2y}{dx^2}\left[= 64 + \frac{3}{4x^4}\right]$ is positive
3(a)	–12 – 69 + 27 + 54 = 0	**B1**	

Question	Answer	Marks	Partial Marks
3(b)	$10x+7=-2x^3+3x^2+33x-5$ oe, soi	**M1**	
	Uses the correct factor $x + 3$ to find a quadratic factor of the polynomial from part (a) oe with at least 2 terms correct	**M1**	
	$-2x^2+9x-4$ or $2x^2-9x+4$	**A1**	
	Factorises or solves *their* 3-term quadratic factor = 0: $(2x-1)(-x+4)$ or $(-2x+1)(x-4)$ or $(2x-1)(x-4)$ oe	**DM1**	**dep** on previous **M1**
	$x = -3$, $x = 0.5$, $x = 4$ nfww	**A1**	**dep** on at least **M0 M1 A1 DM1** awarded
	$A(-3, -23)$, $B(0.5, 12)$, $C(4, 47)$ oe **and** correct method to show mid-point e.g.: $\left(\frac{-3+4}{2}, \frac{-23+47}{2}\right)=\left(\frac{1}{2}, 12\right)$ oe or $\left[\overrightarrow{AB}=\right]\begin{pmatrix}0.5\\12\end{pmatrix}-\begin{pmatrix}-3\\-23\end{pmatrix}=\begin{pmatrix}3.5\\35\end{pmatrix}$ and $\left[\overrightarrow{BC}=\right]\begin{pmatrix}4\\47\end{pmatrix}-\begin{pmatrix}0.5\\12\end{pmatrix}=\begin{pmatrix}3.5\\35\end{pmatrix}$ oe OR [x-coordinate mid-point] $\frac{-3+4}{2}=\frac{1}{2}$ oe **and** valid comment e.g. The points are collinear [so B is the mid-point of AC].	**B2**	**dep** on $x = -3$, $x = 0.5$, $x = 4$ nfww **B1 dep** on $x = -3$, $x = 0.5$, $x = 4$ nfww for $A(-3, -23)$, $B(0.5, 12)$, $C(4, 47)$ oe or [x-coordinate of the mid-point] $\frac{-3+4}{2}=\frac{1}{2}$ oe

Question	Answer	Marks	Partial Marks
4	$\frac{dy}{dx} = -\sec^2(1-x)$ oe	**B2**	**must be seen** **B1** for $\frac{dy}{dx} = k\sec^2(1-x)$ oe, $k \neq -1$ or **SC1** for $\frac{dy}{dx} = -\sec^2 1-x$ or $\frac{dy}{dx} = -\sec^2(1-x)+c$
	Solves $3 = 2 + \tan(1-x)$ as far as $1 - x = \tan^{-1} 1$	**M1**	
	$1-x = \frac{\pi}{4}$ isw or 0.7853[98…] $x = 1-\frac{\pi}{4}$ isw or 0.2146[01…]	**A1**	
	Correct use of chain rule **and** correctly writes in terms of cosine or tangent: $\frac{their(-1)}{\cos^2\left(their\frac{\pi}{4}\right)} \times 0.04$ oe, soi or $their(-1)\left\{1+\tan^2\left(their\frac{\pi}{4}\right)\right\} \times 0.04$ oe soi	**M1**	**dep** on at least **B1** and an attempt to solve $3 = 2 + \tan(1-x)$
	–0.08 oe, nfww	**A1**	**dep** on all previous marks awarded
5(a)	$\lg P = \lg A + T\lg b$ oe nfww **and** correct comparison with $y = mx + c$ soi	**B2**	Must be seen and not from wrong working **B1** for $\lg P = \lg A + T\lg b$ isw, nfww
5(b)	$A = 10^6$ oe isw and $b = 10^{\frac{3}{7}}$ oe isw	**4**	**B2** for $A = 10^6$ oe isw **or B1** correct method which could be used to find A e.g. $\lg A = 6$ or $12 = \frac{3}{7} \times 14 + \lg A$ **B2** for $b = 10^{\frac{3}{7}}$ oe isw or **B1** correct method which could be used to find b e.g. $\lg b = \frac{12-6}{14-0}$ oe or $12 = 14\lg b + 6$

Question	Answer	Marks	Partial Marks
5(c)	lg P_1 = 8 and lg P_2 = 9 soi leading to T_1 = 4.6 to 4.8 or T_2 = 6.8 to 7.2	M1	If graph not used then allow **M1** for substitution of *their A* and *their b* in the exponential equation as far as $\frac{10^8}{theirA} = (theirb)^T$ and $\frac{10^9}{theirA} = (theirb)^T$ OR substitution of *their A* and *their b* or *their* lg*A* and *their* lg *b* in the log equation $\lg 10^8 = their \lg A + T(their \lg b)$ or better and $\lg 10^9 = their \lg A + T(their \lg b)$ or better
	Difference of correct times: $T_2 - T_1$ where T_2 = 6.8 to 7.2 T_1 = 4.6 to 4.8	M1	
	Answer in range 2.2 to 2.4 nfww	A1	
	Alternative method		
	Change in $T = \frac{9-8}{\frac{3}{7}}$	(M2)	**M1** for $\lg 10^8 = 8$ and $\lg 10^9 = 9$ **and** $\frac{\text{Change in } \lg P}{\text{Change in } T} = \frac{3}{7}$
	Answer in range 2.2 to 2.4 nfww	(A1)	
6(a)(i)	$1 + \frac{5}{7}x + \frac{10}{49}x^2$	B2	**B1** for any two correct terms or for the three terms listed but not summed
6(a)(ii)	12	B2	**B1** for 7*n* + 5 = 89 or $7\left(n + \frac{5}{7}\right) = 89$ oe or ${}^nC_1 = 12$
6(b)	${}^8C_4 \times k^4 \times (-2)^4$ [$\times x^4$] oe or $1120k^4$ [x^4] soi	B1	
	${}^8C_2 \times k^6 \times (-2)^2$ [$\times x^2$] oe or $112k^6$ [x^2] soi	B1	
	$\frac{1120k^4}{112k^6} = \frac{5}{8}$ or $\frac{70 \times 16 \times k^4}{28 \times 4 \times k^6} = \frac{5}{8}$ oe, soi	M1	**FT** providing at least **B1** awarded and correct terms attempted
	$k^2 = 16$ soi	A1	
	[For coefficient of *x* to be positive $k < 0$, therefore] $k = -4$	A1	

Question	Answer	Marks	Partial Marks
7(a)	$\dfrac{10(4x-2)^4}{\sqrt{3+(4x-2)^5}}$ or $\dfrac{10(4x-2)^4}{\left(3+(4x-2)^5\right)^{\frac{1}{2}}}$ isw	3	**B2** for correct unsimplified form e.g. $\dfrac{1}{2}\left(3+(4x-2)^5\right)^{-\frac{1}{2}}\times 5\left(4x-2\right)^4\times 4$ or $10\left(3+(4x-2)^5\right)^{-\frac{1}{2}}\left(4x-2\right)^4$ or **B1** for $5(4x-2)^4\times 4$ soi or $\dfrac{1}{2}\left(3+(4x-2)^5\right)^{-\frac{1}{2}}\times \mathrm{g}(x)$
7(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{5(3x+2)-3(5x)}{(3x+2)^2}$ oe isw or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=5x\left(-\left(3x+2\right)^{-2}\times 3\right)+5\left(3x+2\right)^{-1}$ oe isw	B1	
	[$y=10$] $x=-0.8$	B1	
	$\dfrac{0.01}{\delta x}=\left(their\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=-0.8}\right)$ oe	M1	**FT** *their x* providing *their* $x\neq 10$ or 0.01and *their* genuine attempt at a derivative using product or quotient rule
	$\dfrac{1}{6250}$ or 0.00016 or 1.6×10^{-4} isw	A1	**dep** on all previous marks awarded
	Alternative method		
	$x=\dfrac{-2y}{3y-5}$ oe	(B1)	
	$\dfrac{\mathrm{d}x}{\mathrm{d}y}=\dfrac{-2(3y-5)-3(-2y)}{(3y-5)^2}$ oe isw or $\dfrac{\mathrm{d}x}{\mathrm{d}y}=-2y\left(-\left(3y-5\right)^{-2}\times 3\right)+(-2)\left(3y-5\right)^{-1}$	(B1)	
	$\dfrac{\delta x}{0.01}=\left(their\left.\dfrac{\mathrm{d}x}{\mathrm{d}y}\right\|_{y=10}\right)$ oe	(M1)	**FT** *their* genuine attempt at a derivative using product or quotient rule
	$\dfrac{1}{6250}$ or 0.00016 or 1.6×10^{-4} isw	(A1)	**dep** on all previous marks awarded
7(c)(i)	$3x^2\ln x + x^3\times\dfrac{1}{x}$ or better, isw	B2	**B1** for (*their* $3x^2$)$\ln x + x^3\times$ (*their* $\dfrac{1}{x}$)

Question	Answer	Marks	Partial Marks
7(c)(ii)	$\frac{x^3 \ln x}{6} + \frac{x^3}{18} + c$ oe, isw	**B3**	must have arbitrary constant **B2** for $\frac{x^3 \ln x}{6} + \frac{x^3}{18}$ or $\frac{x^3 \ln x}{6} + \frac{kx^3}{3} + c$, $k > 0$ nfww or **B1** for $\frac{1}{6}\int\left(3x^2 \ln x + x^2\right)\mathrm{d}x + \frac{1}{6}\int x^2\,\mathrm{d}x$ soi or $\frac{x^3 \ln x}{6} + \int \frac{x^2}{6}\mathrm{d}x$ soi or $\int\left(3x^2 \ln x\right)\mathrm{d}x = x^3 \ln x - \int x^2\,\mathrm{d}x$ soi or $\int\left(3x^2 \ln x\right)\mathrm{d}x = x^3 \ln x - \frac{x^3}{3}$ soi
8	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{4}\sin\frac{x}{4}$	**M2**	**M1** for $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sin\frac{x}{4}$, $k < 0$ or $k = \frac{1}{4}$
	$y = 0.5$	**B1**	
	$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{4\pi}{3}} = -\frac{1}{4} \times \frac{\sqrt{3}}{2}$ or $-\frac{\sqrt{3}}{8}$	**M1**	**FT** (*their k*) $\times \frac{\sqrt{3}}{2}$ providing at least **M1** awarded
	$[m_{\text{normal}} =] \frac{8}{\sqrt{3}}$ soi	**M1**	**FT** $\frac{-1}{\textit{their}\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=\frac{4\pi}{3}}}$
	$y - 0.5 = \frac{8}{\sqrt{3}}\left(x - \frac{4\pi}{3}\right)$ oe	**A1**	**dep** on previous **M1**; must have exact values **FT** *their* m_{normal} and *their* 0.5 providing both are non-zero, exact values
	$\left(\frac{4\pi}{3} - \frac{\sqrt{3}}{16}, 0\right)$ or exact equivalent; mark final answer	**A1**	

Question	Answer	Marks	Partial Marks
9	$\int e^{\frac{t}{4}}\,dt = 4e^{\frac{t}{4}}(+c)$ and $\int \frac{16e}{t^2}\,dt = \frac{-16e}{t}(+c)$	B3	**B2** for either correct or **B1** for $\int e^{\frac{t}{4}}\,dt = ae^{\frac{t}{4}}(+c)$ where a is a constant, $a > 0$ or $\int \frac{16e}{t^2}\,dt = \frac{-b}{t}(+c)$ where b is a constant $b > 0$
	Correct plan: $\int_0^4 e^{\frac{t}{4}}\,dt + \int_4^k \frac{16e}{t^2}\,dt = 13.4$ soi	M1	
	Correct equation: $4e^1 - 4e^0 + \left(-\frac{16e}{k} + \frac{16e}{4}\right) = 13.4$ OR [When $t = 4$ $s = 4e - 4$ When $t = k$ $s = \frac{-16e}{k} + 8e - 4$ and] $13.4 = \frac{-16e}{k} + 8e - 4$	A1	**dep** on **B3**; implies **M1**
	$k = 10$ or awrt 10.0	A1	dep on all previous marks awarded
10	$\lambda\left(\mathbf{c} + \frac{2}{5}\mathbf{b}\right)$ isw	B2	**B1** for $\lambda(\mathbf{c} + k\mathbf{b})$ where $k \neq \frac{2}{5}$ or 1, $k > 0$
	$2\mathbf{c} + \mu(\mathbf{b} - \mathbf{c})$ isw or $\mu\mathbf{b} + (2 - \mu)\mathbf{c}$ isw or $\mathbf{c} + \mathbf{b} + (1 - \mu)(\mathbf{c} - \mathbf{b})$ isw	B2	**B1** for any of the following with $n > 0$ $2\mathbf{c} + \mu(\mathbf{b} - n\mathbf{c})$ or $n\mathbf{c} + \mu(\mathbf{b} - \mathbf{c})$ or $\mathbf{c} + \mathbf{b} + (1 - \mu)(n\mathbf{c} - \mathbf{b})$ or $n(\mathbf{c} + \mathbf{b}) + (1 - \mu)(\mathbf{c} - \mathbf{b})$
	Equates components at least once $\lambda = 2 - \mu$ or $\frac{2}{5}\lambda = \mu$ soi	M1	**FT** providing of equivalent forms e.g.: $\lambda(s\mathbf{c} + t\mathbf{b})$ and $x\mathbf{c} + \mu(y\mathbf{b} + z\mathbf{c})$ where *s*, *t*, *x*, *y*, *z* are scalars
	$\lambda = 2 - \mu$ **and** $\frac{2}{5}\lambda = \mu$ soi, nfww	A1	
	$\mu = \frac{4}{7}$ $\left[\lambda = \frac{10}{7}\right]$	A1	
	[AE : EB =] 4 : 3 oe	A1	must have earned all previous marks